

Subject: Practical implication of transitory nature of N_{prime} modulo 3 residual classes

Fellow colleagues,

with great delight have I read the article of Lemke Oliver & Soundararajan [1] which indicates an existence of certain bias in the realm of prime numbers.

But sometimes it is the case that too much specialization disallows one to see the forest because of the trees. Hence, authors aimed for analytic explanations there, where deeper empiric inspection of one concrete case could potentially turn out to be equally instructive.

The case we have on our mind is that of *prime modulo 3* residual classes (RCs). For 3 is the only divisor which yields only two RCs: 1 and 2. Thus, only two possible RC transitions are possible in a consecutive pair (CP) of primes P_x and P_{x+1} : 2 \rightarrow 1 transition and 1 \rightarrow 2 transition. Besides this, only two other RC combinations exist for any possible CP: 1 \rightarrow 1 when both CP members belong to RC_1 and 2 \rightarrow 2 when $P_x \bmod 3 == P_{x+1} \bmod 3 == 2$.

Note the symmetry: two RC transitions and two "non-transition" states. *Such symmetry exists, ex vi termini, only in the realm of modulo 3.*

Focusing solely on this transition / non-transition properties of consecutive pairs occurrent among first 30 million primes, one can observe:

- there are 16687076 transitions
- there are 13312923 non-transitions
- longest uninterrupted sequence of *consecutive transitions* consists of 32 primes
- longest uninterrupted sequence of *consecutive non-transitions* consists of 19 primes
- etc.

Those unafraid of induction could thus simply conjecture that given $16687076 / 13312923 \approx 1.253$, **it is approx. 25% more probable that P_{x+1} will belong to different modulo3RC than P_x .**

In other terms: *approximately 25% carbon dioxide less could be potentially emitted* if machines aiming to discover new prime P_{x+1} would explore:

- sequences $(P_x + 2 + 3*n)$ if it is known that $P_x \bmod 3 == 1$
- sequences $(P_x + 4 + 3*n)$ if it is known that $P_x \bmod 3 == 2$

It is in this point that we disagree with the expression "no practical use" contained in the statement : *"this 'anti-sameness' bias has no practical use or even any wider implication for number theory, as far as Soundararajan and Lemke Oliver know"* recently published in Your journal [2].

With highest regards,
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[1] Robert J. Lemke Oliver, Kannan Soundararajan. Unexpected biases in the distribution of consecutive primes. 2016. <http://arxiv.org/abs/1603.03720>

[2] Evelyn Lamb. Peculiar pattern found in 'random' prime numbers. 2016. <http://www.nature.com/news/peculiar-pattern-found-in-random-prime-numbers-1.19550>

[3] https://en.wikipedia.org/wiki/Monty_Hall_problem